Alg. 2 Warm Up #4-5 Quiz first.

Solve:

1. $\log_3\left(\frac{1}{27}\right) = x$

2. $3|2x - 5| + 7 \leq 25$

Simplify:

3. $(6x^4y^{-3})^{-2}$

4. $\left[\frac{4x^4}{(5xy)^2}\right]^{1/2}$

Look over your Chapter 5 test.

Let me know if you have questions.

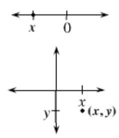
Put it in the basket on my desk and get ready for your quiz.

Pencil only.

METHODS AND MEANINGS p. 263

MATH NOTES

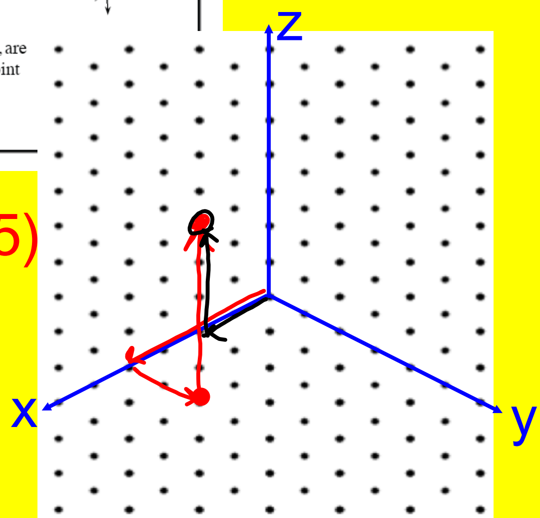
Locating Points in Three Dimensions



When locating a point on a *number line*, a single number, x, is used.

The location of a point in a *plane* is given by two numbers, (x, y), called an ordered pair.

To locate a point in *space*, three numbers, (x, y, z), are used, which are called an **ordered triple**. The point (2, 3, 1) is shown at right. The dotted lines help clarify which coordinate was graphed.



Example: (4, 2, 5)

(2, 0, 3)

lands in the same place, so you need the arrows!

METHODS AND MEANINGS p. 271

MATH NOTES

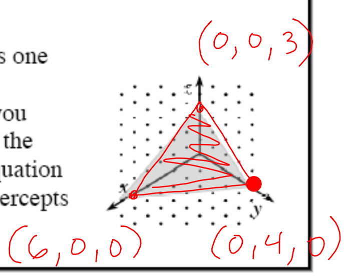
Graphing Planes in Three Dimensions

To graph a plane, it is easiest to use the intercepts to draw the "trace lines" (the intersections of the plane with the xy-, xz-, and yz-planes) that will represent the plane.

To find the intercepts, let two of the variables equal zero. Then solve to find the intercept corresponding to the remaining variable.

For example, for $2x+3y+4z=12$, the x-intercept is found by letting y and z equal zero, which gives $2x=12$. Therefore the x-intercept is (6, 0, 0). Similarly, the y-intercept is (0, 4, 0), and the z-intercept is (0, 0, 3).

Drawing the line between two intercepts gives one of the trace line in that coordinate plane. For example, connecting the x- and y-intercepts, you would get the equation $2x+3y=12$, which is the trace line in the xy-plane when $z=0$ in the equation $2x+3y+4z=12$. Connecting the x- and z-intercepts gives the trace line in the xz-plane, etc.



HW Questions:

6-80. Solve the system of equations at right and then check your solution in each equation. Be sure to keep your work well organized.

$$\begin{aligned} ①\quad & x+2y-z=-1 \\ ②\quad & (2x-y+3z=13)(2) \\ ③\quad & x+y+2z=14 \end{aligned}$$

Eliminate y

$② + ③ \rightarrow 3x + 5z = 27$

$① + 2② \rightarrow -(5x + 5z = 25)$

$-2x = 2$

$x = -1$

$$\begin{aligned} 4x-2y+6z&=26 \\ x+2y-z&=-1 \\ \hline 5x \quad +5z&=25 \end{aligned}$$

plug in & find z,
then go back to one
of the original equations
to find y.

$\boxed{(-1, \quad , \quad)}$

6-81. Find an equation for the parabola that passes through the points $(-1,10)$, $(0,5)$, and $(2,7)$.

$y = ax^2 + bx + 5$

$c = 5$

$(-1,10) \rightarrow 10 = a(-1)^2 + b(-1) + 5 \rightarrow a - b = 5$

$(2,7) \rightarrow 7 = a(2)^2 + b(2) + 5 \rightarrow 4a + 2b = 2$

$2a + b = 1$

Simplify and solve your 2 equations
for a and b

6-82. Change each of the following equations from logarithmic form to exponential form, or vice versa.

a. $a = \log_b 24$

b. $3x = \log_{2y} 7$

c. $3y = 2^{5x}$ $\log_2 3y = 5x$

base

exponent

d. $4p = (2q)^6$

6-83. Add or subtract and simplify each of the following expressions. Justify that each step of your process makes sense. LCD $= (x-1)(x-2)$

a. $\frac{3x}{x^2+2x+1} + \frac{3}{x^2+2x+1}$

b. $\frac{3}{x-1} - \frac{2}{x-2}$

$$\frac{(x-2)}{(x-2)} \cdot \frac{3}{(x-1)} - \frac{2}{(x-2)} \frac{(x-1)}{(x-1)}$$

$$\frac{3x-6-2x+2}{(x-1)(x-2)}$$

6-84. On their Team Test, Raymond, Sarah, Hannah, and Aidan were given $y = 4x^2 - 24x + 7$ to change into graphing form. Raymond noticed that the leading coefficient was a 4 and not a 1. His team agreed on a way to start rewriting, but then they worked in pairs and got two different solutions, shown below.

Raymond and Hannah

(1) $y = 4x^2 - 24x + 7$

(2) $y = 4(x^2 - 6x) + 7$

(3) $y = 4(x^2 - 6x + 3^2) + 7 - 36$

(4) $y = 4(x-3)^2 - 29$

Aidan and Sarah

(1) $y = 4x^2 - 24x + 7$

(2) $y = 4(x^2 - 6x + 9) + 7 - 9$

(3) $y = 4(x-3)^2 + 7 - 3^2$

(4) $y = 4(x-3)^2 - 2$

Hannah says, *"Aidan and Sarah made a mistake in Step 3. Because of the factored 4 they really added 4(9) to complete the square, so they should subtract 36, not just 9."* Is Hannah correct? Justify your answer by showing whether the results are equivalent to the original equation.

6-85. Use the correct method from problem 6-84 to change each of the following equations to graphing form. Then, without graphing, find the vertex and equation of the line of symmetry for each.

a. $y = 2x^2 - 8x + 7$

b. $y = 5x^2 - 10x - 7$

$y = 2(x^2 - 4x + 4) + 7 - 8$

$y = 2(x-2)^2 - 1$

V: $(2, -1)$

axis

$x = 2$

$5(x^2 - 2x + 1) - 7 - 5$

$y = 5(x-1)^2 - 12$

6-86. Shift the graph of $\log x$ up 3 units and to the right 6. Graph both the original and the transformed graph on the same set of axes and write the equation for the transformed graph.

$y = \log_{10}(x-6) + 3$

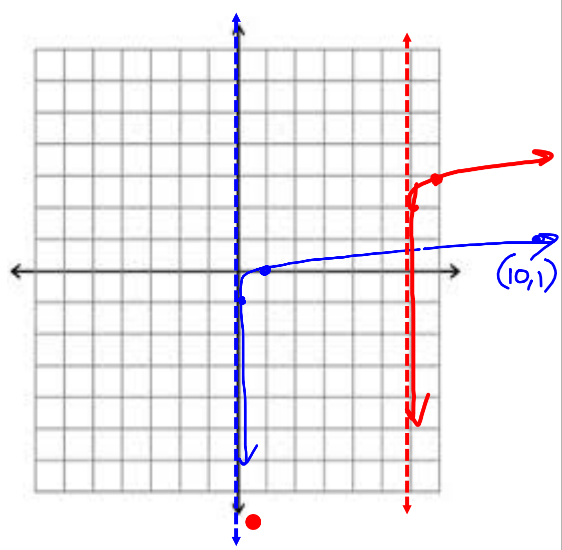
parent

$y = \log_{10} x$

x	y
$\frac{1}{10}$	-1
1	0
10	1

Rt 6 up 3

x	y



6-87. Given $f(x) = 2x^2 - 4$ and $g(x) = 5x + 3$, find the value of each expression below.

a. $f(a)$ b. $f(3a)$ c. $f(a+b)$

d. $f(x+7)$ e. $f(5x+3)$ f. $g(f(x))$

c) $f(a+b)$

$= 2(a+b)^2 - 4$

$= 2(a^2 + 2ab + b^2) - 4$

$= 2a^2 + 4ab + 2b^2 - 4$

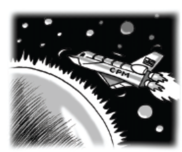
$g(f(x)) = 5(2x^2 - 4) + 3$

$= 10x^2 \underline{- 20 + 3}$

$g(f(x)) = 10x^2 - 17$

Yesterday's CP's: 6 - # 66 ---> 68, 70

6-66. CPM engineers are considering developing a private space rocket. In a computer simulation, the rocket is approaching a star and is caught in its gravitational pull. When the rocket's engines are fired, the rocket will slow down, stop momentarily, and then pick up speed and move away from the star, avoiding its gravitational field. CPM engaged the rocket engines when it was 750 thousand miles from the star. After one full minute, the rocket was 635 thousand miles from the star. After two minutes, the ship was 530 thousand miles from the star.

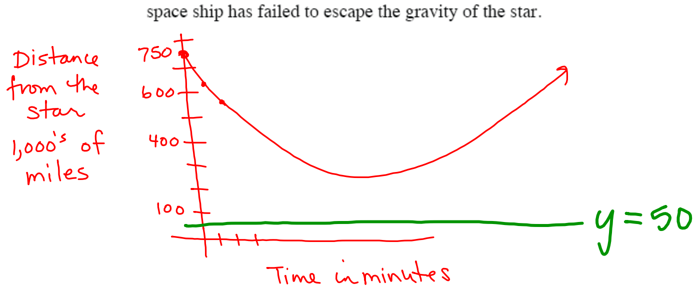


a. Name the three points given in the information above if x = the time since the engines were engaged and y = the distance (in thousands of miles) from the star. (0, 750) (1, 635) (2, 530)

b. Based on the points in part (a), make a rough sketch of a graph that shows the distance reaching a minimum and then increasing again, over time. What kind of function could follow this pattern? $y = ax^2 + bx + c$

c. Find the equation of a graph that fits the three points you found in part (a).

d. If the ship comes within 50 thousand miles of the star, the shields will fail and the ship will burn up. Use your equation to determine whether the space ship has failed to escape the gravity of the star.



(6c) $y = ax^2 + bx + c$

$(0, 750) \longrightarrow c = 750$

$(1, 635)$ $\quad 635 = a(1)^2 + b(1) + 750$

$-750 \qquad -750$

$-115 = a + b$

$(2, 530)$ $\quad 530 = a(2)^2 + b(2) + 750$

$-750 \qquad -750$

$\dfrac{-220 = 4a + 2b}{-2} \rightarrow$

$-115 = a + b$

$110 = -2a - b$

6-67. Sickly Sid has contracted a serious infection and has gone to the doctor for help. The doctor takes a blood sample and finds 900 bacteria per cc (cubic centimeter) and gives Sid a shot of a strong antibiotic. The bacteria will continue to grow for a period of time, reach a peak, and then decrease as the medication succeeds in overcoming the infection. After ten days, the infection has grown to 1600 bacteria per cc. After 15 days it has grown to 1875.

peak

$(0, 900) \rightarrow \boxed{c = 900}$ $\quad (10, 1600) \quad (15, 1875)$

a. Name three data points given in the problem statement.

b. Make a rough sketch that will show the number of bacteria per cc over time. Must have appropriate scale and axes labeled!

c. Find the equation of the parabola that contains the three data points.

d. Based on the equation, how long will it take until the bacteria are eliminated? (y = 0 and solve for x. Interpret your answer in context!)

e. Based on the equation, how long had Sid been infected before he went to the doctor?

$y = ax^2 + bx + 900$

6-70. Make a conjecture about how you would find the equation of a cubic function that passes through a given set of points when graphed, $y = ax^3 + bx^2 + cx + d$. How many points do you think you would need to be given to be able to determine a unique equation? How could you extend the method you developed for solving a quadratic to solving a cubic?

Week 4 Classwork

Warm up

6 - # 16 ---> 20
(iso graph for # 18, 19ab, 20c)

6 - # 30 ---> 33, 35
(iso graphs for # 31, 32, 35)

6 - # 44 ---> 48
(iso graph for # 45)

6 - # 60, 61, 64, 65

6 - # 66 ---> 68, 70

CP's: 6- #88 ---> 93

6.2.1 How can I solve exponential equations?

Using Logarithms to Solve Exponential Equations

In Chapter 5, you learned what a logarithm was and several important facts about logs. In this lesson, you will learn about a property of logarithms that will be very useful for solving problems that involve exponents.

6-88. LOGARITHMS SO FAR

There are three important log facts you have learned so far. Discuss these questions with your team to ensure everyone remembers these ideas. For each problem, make up an example to illustrate your ideas.

a. What is a logarithm? How can log equations be converted into another form?

b. What do you know about the logarithm key on your calculator?

c. What does the graph of $y = \log(x)$ look like? Write a general equation for $y = \log(x)$.

6-89. Marta was convinced that there had to be some way to graph $y = \log_2 x$ on her graphing calculator. She typed in $y = \log(2^x)$ and pressed GRAPH.

"It WORKED!" Marta yelled in triumph.

"Whaaaat?" said Celeste. *"I think* $y = \log_2 x$ *and* $y = \log(2^x)$ *are totally different, and I bet we can show it by converting both of them to exponential form."*

"Yeah, I think you're wrong, Marta," said Sophia. *"I think we can show that* $y = \log_2 x$ *and* $y = \log(2^x)$ *are totally different by looking at the graphs."*

a. Show that the two equations are different by sketching the graph of $y = \log_2 x$. Then sketch what your graphing calculator shows to be the graph of $y = \log(2^x)$.

b. Now show that $y = \log_2 x$ and $y = \log(2^x)$ are different by converting both of them to exponential form.

6-90. The work you did in problem 6-89 is a **counterexample**, which shows that in general, the statement $\log_b x = \log(b^x)$ is *false*. For each of the following log statements, use the strategies from problem 6-89 to determine whether they are true or false, and justify your answer. Be ready to present your conclusions and justifications.

a. $\log_5(25) \overset{?}{=} \log_{25}(5)$

b. $\log(x^2) \overset{?}{=} (\log x)^2$

c. $\log(7^x) \overset{?}{=} x\log(7)$

d. $\log(2x) \overset{?}{=} \log_2 x$

6-91. In the previous problem only *one* of the statements was true. c $\log 7^x = x\log 7$

a. Use different numbers to make up four more statements that follow the same pattern as the one true statement, and test each one to see whether it appears to be true.

b. Use your results to complete the following statement, which is known as the **Power Property of Logarithms**: $\log(b^x) =$ $x(\log b)$

$y = \log b^x$

let $\begin{cases} b = 2 \\ x = 3 \end{cases}$

$\log 2^3 \overset{?}{=} 3(\log 2)$

$\approx 0.90 \qquad \approx 0.90$

let $\begin{cases} b = 12 \\ x = 2 \end{cases}$

$\log 12^2 \overset{?}{=} 2(\log 12)$

$\approx 2.16 \qquad \approx 2.16$

6-92. Do you remember solving problems like $1.04^x = 2$ in your homework? What method(s) did you and your teammates use to find x? In tonight's homework there are several more of these problems. (You probably wish there were a more efficient way!)

$$(1.04)^x = 2$$

$$x$$

$$(1.04)^1 \quad 1.04$$

$$1.04 \quad 2$$

6-93. THERE MUST BE AN EASIER WAY

It would certainly be helpful to have an easier method than guess and check to solve equations like $1.04^x = 2$. Complete parts (a) through (c) below to discover an easier way.

a. What makes the equation $1.04^x = 2$ so hard to solve? variable in exponent can't make bases match

b. Surprise! In the first part of this lesson, you already found a method for getting rid of inconvenient exponents! Talk with your team about how your results from problems 6-90 and 6-91 can help you rewrite the equation $1.04^x = 2$. Be prepared to share your ideas with the class.

c. Solve $1.04^x = 2$ using this new method. Be sure to check your answer.

Take $\log_{10}$ of both sides.

$$\log 1.04^x = \log 2$$

$$\frac{x(\cancel{\log 1.04})}{\cancel{\log 1.04}} = \frac{\log 2}{\log 1.04}$$

$$x \approx 17.67$$

check:

$$1.04^{17.67} \approx 2$$

HW: 6- # 95 ---> 103